

Vacuum-driven Metamorphosis

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We show that nonperturbative vacuum effects can produce a vacuum-driven transition from a matter-dominated universe to one in which the effective equation of state is that of *radiation* plus cosmological constant. The actual material content of the universe after the transition remains that of non-relativistic matter. This metamorphosis of the equation of state can be traced to nonperturbative vacuum effects that cause the scalar curvature to remain nearly constant at a well-defined value after the transition, and is responsible for the observed acceleration of the recent expansion of the universe.

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I. INTRODUCTION

Recent observations of high-redshift Type Ia supernovae (SNe-Ia) [1,2] indicate that the expansion of the universe is accelerating at the present time. Attempts to explain this effect include a revival of the cosmological constant term in Einstein's equations [3,4] as well as the hypothesized existence of a classical scalar field with a potential term (quintessence [5]). In previous work [6,7], we show that a free quantized massive scalar field can account for the SNe-Ia observations in a new way. Non-perturbative vacuum contributions to the effective action of such a field [8] lead to a cosmological solution in which the scalar curvature has a constant value after a cosmic time t_j that depends on the mass of the scalar field and its curvature coupling. This solution implies an accelerating universe at the present time and gives a good fit to SNe-Ia data for a particle mass parameter of about 10^{-33} eV, without the necessity of introducing a cosmological constant term into Einstein's equations. Such a low mass parameter may arise from pseudo Nambu-Goldstone bosons [9]. A graviton of such a small non-zero mass would be expected to produce similar effects.

In Ref. [7], we also show that our cosmological model gives light-element abundances in agreement with standard big-bang nucleosynthesis and that it is consistent with present data on the small angular scale fluctuations of the cosmic microwave background radiation (CMBR), which tend to favor spatial flatness. Our model does not suffer from a fine-tuning problem because, for the allowed range of the mass parameter, the probability is high that the matter and vacuum energy densities are of the same order of magnitude at the present time.

The main point of this letter is to elucidate a remarkable feature of the equation of state of our model,

namely, that the equation of state undergoes a *metamorphosis* from an equation of state dominated by pressureless matter (without a cosmological constant) to an effective equation of state that can be described in classical terms as that of mixed radiation and cosmological constant. As we show, this metamorphosis is initiated by the quantum vacuum terms which grow rapidly after time t_j and combine with pressureless matter to give an effective stress tensor identical to that of radiation plus cosmological constant.

In the next section we summarize the main features of our model, and in Section III we analyze the equation of state. Our conclusions are given in Section IV.

II. SUMMARY OF OUR MODEL

We consider a free, massive quantized scalar field of inverse Compton wavelength m , and curvature coupling ξ . The effective action for gravity coupled to such a field is obtained by integrating out the vacuum fluctuations of the field [6,8]. This effective action is the simplest one that gives the standard trace anomaly in the massless-conformally-coupled limit, and contains the nonperturbative sum (in arbitrary dimensions) of *all* terms in the propagator having at least one factor of the scalar curvature, R . The trace of the Einstein equations, obtained by variation of this effective action with respect to the metric tensor takes the following form in a Friedmann-Robertson-Walker (FRW) spacetime (in units such that $c = 1$), with *zero* cosmological constant [6,7]:

$$R + \frac{T_{cl}}{2\kappa_o} = \frac{\hbar m^2}{32\pi^2\kappa_o} \left\{ (m^2 + \bar{\xi}R) \ln |1 + \bar{\xi}Rm^{-2}| - \frac{m^2\bar{\xi}R}{m^2 + \bar{\xi}R} \left(1 + \frac{3}{2}\bar{\xi} \frac{R}{m^2} + \frac{1}{2}\bar{\xi}^2 \frac{R^2}{m^4} (\bar{\xi}^2 - (1080)^{-1}) + v \right) \right\}, \quad (1)$$

where T_{cl} is the trace of the stress tensor of classical, perfect fluid matter, $\kappa_o \equiv (16\pi G)^{-1}$ (G is Newton's con-

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stant), $\bar{\xi} \equiv \xi - 1/6$, and $v \equiv (R^2/4 - R_{\mu\nu}R^{\mu\nu})/(180m^4)$ is a curvature invariant that vanishes in de Sitter space.

As noted earlier, m is the inverse Compton wavelength of the field. It is related to the actual mass of the field by $m_{\text{actual}} = \hbar m$. Equation (1) above is nonperturbative in R because it contains terms that involve an infinite sum of powers of R . However, for a sufficiently low mass, it is possible to treat $m_{\text{actual}}^2/m_{Pl}^2 \equiv \hbar m^2/(16\pi\kappa_o)$ (where m_{Pl} is the Planck mass) as a small parameter and expand perturbatively in this parameter, as we do in obtaining the solution described below.

In Refs. [6] and [7], we show that, for a sufficiently low mass, in an expanding FRW universe the quantum contributions to the Einstein equations become significant at a time t_j , when the density of classical matter, ρ_m , has decreased to a value given by

$$\rho_m(t_j) = 2\kappa_0 \bar{m}^2, \quad (2)$$

where

$$\bar{m}^2 \equiv m^2/(-\bar{\xi}). \quad (3)$$

We find that the time t_j occurs in the matter-dominated stage of the evolution. Furthermore for $t > t_j$ we find that the scalar curvature, R , remains constant to excellent approximation at the value \bar{m}^2 . For $t < t_j$, the quantum contributions to the Einstein equations are negligible and the scale factor is that of a matter-dominated FRW universe. Then, Eq. (2) implies that, in a spatially flat universe with line element $ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$, one has

$$t_j = (2/\sqrt{3})\bar{m}^{-1}, \quad H(t_j) = \bar{m}/\sqrt{3}, \quad (4)$$

where $H(t_j)$ is the Hubble constant at t_j .

As shown in [7], the condition of the constancy of the scalar curvature after t_j leads to a solution for the scale factor that can be joined, with continuous first and second derivatives (i.e., in a C^2 manner), to the matter-dominated solution for $t < t_j$. The scale factor is then given by

$$\begin{aligned} a(t) &= a(t_j) \sqrt{\sinh\left(\frac{t\bar{m}}{\sqrt{3}} - \alpha\right) / \sinh\left(\frac{2}{3} - \alpha\right)}, \quad t > t_j, \\ &= a(t_j) \left(\sqrt{3}\bar{m}t/2\right)^{2/3}, \quad t < t_j, \end{aligned} \quad (5)$$

with

$$\alpha = 2/3 - \tanh^{-1}(1/2) \simeq 0.117. \quad (6)$$

We find that the above solution also satisfies, up to terms of order m_{actual}/m_{Pl} , the one remaining independent Einstein equation in a FRW universe, which can be taken to be the time-time component $G_{00} = (2\kappa_o)^{-1}T_{00}$, where $G_{\mu\nu}$ is the Einstein tensor. This equation takes the form, with zero cosmological constant,

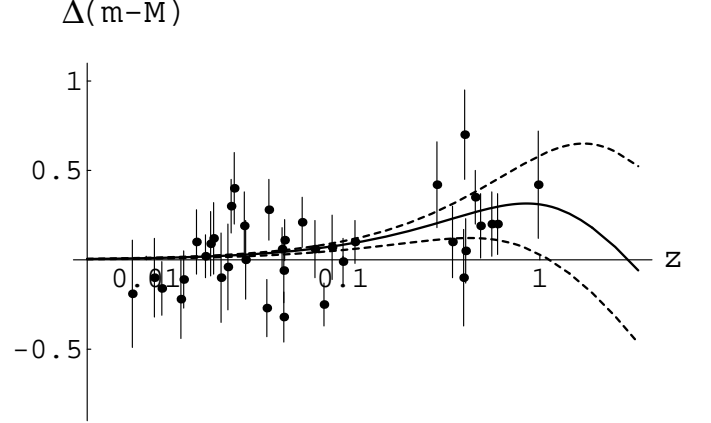


FIG. 1. A plot of the difference between apparent and absolute magnitudes, as functions of redshift z , normalized to an open universe with $\Omega_0 = 0.2$ and zero cosmological constant. The points with vertical error bars represent SNe-Ia data obtained from Ref. [2]. The two dashed curves represent the values (a) $\bar{m}/h = 6.40 \times 10^{-33}$ eV (lower dashed curve), and (b) $\bar{m}/h = 7.25 \times 10^{-33}$ eV (upper dashed curve). The solid curve represents the intermediate value (c) $\bar{m}/h = 6.93 \times 10^{-33}$ eV.

$$\begin{aligned} k_o G_{00} &= \frac{1}{2}\rho_m - \frac{\hbar}{64\pi^2} \left\{ \frac{\bar{\xi} R_{00}}{m^2 + \bar{\xi} R} (m^4 + 2m^2 \bar{\xi} R \right. \\ &\quad + \frac{R_{\alpha\beta} R^{\alpha\beta}}{90} + R^2 \left(\bar{\xi}^2 - \frac{1}{270} \right) \Big) - 3\bar{\xi}^2 R R_{00} \\ &\quad + m^2 \bar{\xi} G_{00} \Big\} - \frac{\hbar}{64\pi^2} \ln \left| 1 + \frac{\bar{\xi} R}{m^2} \right| \left\{ -\frac{m^4 g_{00}}{2} \right. \\ &\quad + 2m^2 \bar{\xi} G_{00} - \frac{g_{00} R^2}{2} \left(\bar{\xi}^2 + \frac{1}{90} \right) \\ &\quad \left. + \frac{1}{90} g_{00} R_{\alpha\beta} R^{\alpha\beta} + 2\bar{\xi}^2 R R_{00} - \frac{2}{45} R_0{}^\alpha R_{0\alpha} \right\}. \end{aligned} \quad (7)$$

To verify that Eq. (5) is indeed a solution of the above equation for $t > t_j$, we note that, when R is very close to the value \bar{m}^2 , the dominant terms in the right hand side of Eq. (7) are those that have a factor of $m^2 + \bar{\xi} R$ in the denominator. Keeping these terms and substituting for the various curvature quantities derived from Eq. (5), it

is straightforward to check that Eq. (5) satisfies Eq. (7) up to terms of order $m_{\text{actual}}/m_{\text{Pl}}$.

The solution in Eq. (5) corresponds to a universe that is accelerating (i.e., has negative deceleration parameter) for $t > \sqrt{3}\bar{m}^{-1}(\alpha + \tanh^{-1}(2^{-1/2})) \simeq 1.50 t_j$. This solution gives a good fit to the SNe-Ia data, for the mass range

$$6.40 \times 10^{-33} \text{ eV} < \left(\frac{\bar{m}}{h}\right) < 7.25 \times 10^{-33} \text{ eV}, \quad (8)$$

where h is the present value of the Hubble constant, measured as a dimensionless fraction of the value 100 km/(s Mpc). In Fig. 1, we give plots from [7] of the difference between apparent and absolute magnitudes of sources as functions of the redshift z . The two dashed curves represent the two extremal values of the range (8). The solid curve represents an intermediate value of m_h , and the data points represent SNe-Ia data acquired from Ref. [2].

In [7], we also show that the ratio of matter density to critical density at the present time, Ω_0 , is a function of the single parameter \bar{m}/h and turns out to have the range $0.58 > \Omega_0 > 0.15$ for the range of values of Eq. (8). For the same range of values, the age of the universe t_0 lies in the range $8.10 h^{-1} \text{ Gyr} < t_0 < 12.2 h^{-1} \text{ Gyr}$.

III. EQUATION OF STATE

Although our model incorporates the effective action of a quantum field, it admits a simple, classical representation. Indeed, the scale factor (5) may be used to find the total effective energy density ρ and pressure p of vacuum plus matter by directly computing the Einstein tensor. We obtain, for $t > t_j$,

$$\begin{aligned} \rho(t) &= 2\kappa_0 G_{00} = (\kappa_0 \bar{m}^2/2) \coth^2 \left(t\bar{m}/\sqrt{3} - \alpha \right) \\ &= (3/2)\kappa_0 \bar{m}^2 (a(t)/a(t_j))^{-4} + (1/2)\kappa_0 \bar{m}^2 \end{aligned} \quad (9)$$

$$\begin{aligned} p(t) &= 2\kappa_0 a(t)^{-2} G_{ii} = (\kappa_0 \bar{m}^2/6) \coth^2 \left(t\bar{m}/\sqrt{3} - \alpha \right) \\ &\quad - (2/3)\kappa_0 \bar{m}^2. \end{aligned} \quad (10)$$

The effective equation of state for $t > t_j$ is therefore

$$p = (1/3)\rho - (2/3)\kappa_0 \bar{m}^2, \quad (11)$$

which is identical to the equation of state for a classical model consisting of radiation plus cosmological constant. In our model the equation of state of pressureless matter and the equation of state of quantum vacuum terms combine in a manner so as to effectively appear as a sum of radiation and cosmological constant equations of state. Our model differs, even at the classical level, from the usual mixed matter-cosmological constant model because (i) for $t < t_j$ the effective cosmological constant vanishes, and (ii) for $t > t_j$ vacuum contributions transmute the effective equation of state into that of *radiation*

(rather than pressureless matter) plus cosmological constant; this surprising metamorphosis is a result of the near-constancy of the scalar curvature, which causes certain terms in $T_{\mu\nu}$ to take the form of an effective cosmological constant term in Einstein's equations. In a *general* spacetime, these terms do *not* have the form of a cosmological constant term.

The equation of state for the quantum vacuum terms *alone* may be inferred from Eqs. (9) and (10), and from the fact that the density of pressureless matter is given by

$$\begin{aligned} \rho_m(t) &= \rho_m(t_j) (a(t_j)/a(t))^3 \\ &= 2\kappa_0 \bar{m}^2 \left(\frac{\sinh(2/3 - \alpha)}{\sinh(t\bar{m}/\sqrt{3} - \alpha)} \right)^{3/2}, \end{aligned} \quad (12)$$

where Eqs. (2) and (5) have been used to arrive at the second equality. The quantum vacuum energy density ρ_V and pressure p_V then follow, for $t > t_j$, as

$$\begin{aligned} \rho_V(t) &= \rho(t) - \rho_m(t) = \frac{\kappa_0 \bar{m}^2}{2} \left[\coth^2 \left(t\bar{m}/\sqrt{3} - \alpha \right) \right. \\ &\quad \left. - 4 \left(\frac{\sinh(2/3 - \alpha)}{\sinh(t\bar{m}/\sqrt{3} - \alpha)} \right)^{3/2} \right] \end{aligned} \quad (13)$$

$$p_V(t) = p(t), \quad (14)$$

with $p(t)$ given by Eq. (10). The above equations show that for $t > t_j$ the vacuum energy density is positive, while the vacuum pressure is negative. As stated earlier, the vacuum terms are negligible for $t < t_j$.

After some straightforward algebra, we find the equation of state for the quantum vacuum terms:

$$\rho_V = 3p_V + 2\bar{m}^2 \kappa_0 \left[1 - (1 + 2p_V/(\kappa_0 \bar{m}^2))^{3/4} \right]. \quad (15)$$

This vacuum equation of state joins continuously to the equation of state $\rho_V = p_V = 0$ at $t = t_j$ and asymptotes to the pure cosmological constant equation of state $\rho_V = -p_V$ as $t \rightarrow \infty$. Equation (15) shows that the quantum vacuum stress tensor in our model is parametrized by the single parameter, \bar{m} , and is different from that of a pure cosmological constant.

IV. CONCLUSIONS

In this letter, we have outlined a cosmological model that includes nonperturbative quantum vacuum effects of a renormalizable free field of very low mass in curved spacetime. The only adjustable parameter, determined by observation, is the mass scale \bar{m} of the proposed particle. With \bar{m} in the range of Eq. (8), the model is in agreement with the observed light-element abundances, CMBR fluctuation spectrum, age of the universe, and the magnitude-redshift relation of Type Ia supernovae.

In terms of the equation of state, our model admits a simple description that could also be a generic feature of theories other than the one considered here. At a time t_j , $m^2 + \bar{\xi}R$ first becomes small and terms in the quantum stress tensor become large, causing the scalar curvature after t_j to take the nearly constant value $\overline{m}^2 = -m^2/\bar{\xi}$ (for $\bar{\xi} < 0$). The constancy of R effectively forces the combined equation of state of quantum vacuum terms and pressureless matter to undergo a metamorphosis to that of radiation plus cosmological constant. This effect explains the observations regarding the acceleration of the recent expansion of the universe. Other mechanisms that cause the scalar curvature to remain constant would give rise to a similar metamorphic behavior of the equation of state at the phenomenological level. Non-perturbative terms in the vacuum stress tensor seem to provide the simplest example of such a mechanism.

Finally, we note that the quantum vacuum terms would also alter density inhomogeneities that existed at time t_j because these terms would remain insignificant in regions having average density at t_j large with respect to $\rho_m(t_j) \equiv 2\overline{m}^2\kappa_o$. This may eventually provide a further observational test of our model.

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- [1] S. Perlmutter *et al.*, *Nature* **391**, 51 (1998).
 - [2] A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).
 - [3] See the review, V. Sahni and A. Starobinsky, astro-ph/9904398, and references therein.
 - [4] See the review, N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, astro-ph/9906463, and references therein.
 - [5] L. Wang, R. R. Caldwell, J. P. Ostriker and P. J. Steinhardt, astro-ph/9901388.
 - [6] L. Parker and A. Raval, gr-qc/9905031 (1999), *Phys. Rev. D* (in press).
 - [7] L. Parker and A. Raval, gr-qc/9908013 (1999).
 - [8] L. Parker and D. J. Toms, *Phys. Rev. D* **31**, 953 (1985); *Phys. Rev. D* **31**, 2424 (1985); I. Jack and L. Parker, *Phys. Rev. D* **31**, 2439 (1985).
 - [9] J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995); V. Sahni and S. Habib, *Phys. Rev. Lett.* **81**, 1766 (1998); M. Fukugita and T. Yanagida, preprint YITP/K-1098 (1994).